An Attractive Financial Tool in Aging Populations: Determining an Optimal Principal Limit Factor for Reverse Mortgages under the Economic-Based Model

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Abstract

The reverse mortgage (RM) is an attractive financial tool governments can use to improve the economic status of their aging populations, because it gives elders greater personal responsibility in covering their living expenses. Determining an optimal principal limit factor (PLF) is very important, since it influences elders' willingness to participate in the RM program, which in turn influences not only the future development of the RM market but also the effects of portfolio diversification on the collateral risk and the longevity risk. The goal of this study was to develop a general economic model for calculating an optimal PLF that includes the longevity risk, the collateral risk and the interest risk. We provide numerical analyses that illustrate this calculation for both uninsured and insured RMs under two general economic assumptions: the lender breaks even and the lender achieves maximum profit. Our sensitivity analyses provide useful information to help policymakers and market participants modulate a reasonable PLF responsive to changes in the economic situation and the relevant risks. Our models and results provide useful information that can help them determine a reasonable PLF and manage risks.

Keywords: Principal Limit Factor, Reverse Mortgage, Crossover Risk, Longevity Risk, Collateral Risk, Interest Risk

1. Introduction

Large-scale demographic changes are taking place in many countries, including China, Japan, Korea, the U.S. and members of the European Union. These countries have rapidly aging populations, which has gradually created social and economic problems (Von Weizsacker, 1996; Davis, 1997; Faruqee and Muhleisen, 2003; Li, 2005). Because house wealth is often the major component of savings for elderly homeowners,[1] effectively putting this wealth to use is important for improving their quality of life. However, elderly homeowners have a strong distaste for moving out of their house (Venti and Wise, 2000; Davidoff and Welke, 2007). Reverse mortgages (hereafter RMs) allow elderly homeowners to obtain loans from lenders (i.e., banks) using their house as collateral. An added bonus is that RM borrowers can still stay in their house. Thus, the RM is an attractive financial tool for borrowers. Moreover, developing the RM market is a good way for governments to solve the problems they face with aging populations, because elders can use this product to take more personal responsibility in covering their living expenses and unexpected bills.

RM borrowers enjoy the following benefits. First, as noted above, RMs allow them to convert their home equity into cash without having to move out of the house. Second, homeowners have no obligation to repay the loan as long as they stay in the house. Finally, the loan repayments are capped at the proceeds from the sale of the property, the so-called non-recourse provision. Economists believe that RMs have the potential to improve the wellbeing of these house-rich but cash-poor elderly homeowners[2] by paying for their long-term

[1] For many elderly homeowners, house wealth is the largest non-pension component of their total wealth. For example, according to the 2004 *Survey of Consumer Finances* (SCF), house wealth constitutes at least 80% of the total wealth of 27.8% of homeowners age 62 or above. In addition, 13.3% of such homeowners have a house-value-to-income ratio of at least 10. For more details, see the *Federal Reserve Bulletin*.

[2] Using data from the English Longitudinal Study of Ageing 2002-2003, Sodha (2005) estimated that 10.2 percent of retirees in Britain had an income below the "modest but adequate" standard (£157 per week before housing costs),

care, assuring their financial independence and maintaining their standard of living while allowing them to continue living in their home. Several studies have demonstrated that RMs indeed improve elders' financial situation (Speare, 1992; Mayer and Simons, 1994b; Rasmussen et al. 1995; Morgan, Megbolugbe and Rasmussen, 1996; Venti and Wise, 2000).

Although RMs have been in existence for several dozen years in the U.S., the growth of the RM market has not been as great as expected. Only about 400,000 of the tens of millions of eligible homeowners in the U.S. took out loans through the Home Equity Conversion Mortgage (hereafter HECM) program[3] from 1989 (the inception of the RM program) to the end of 2007 (Bishop and Hui, 2008). Only about 1.4 percent of homeowners had active RMs in 2009 (Merrill et al., 1994).

Academic studies have suggested a number of explanations for the small size of the RM market: the high cost of participation in the RM program (e.g., origination fee, mortgage insurance premium, interest), regulatory and legal barriers, moral hazard and adverse selection, financial awareness and literacy, the perception of house equity as a safety for large medical expenses, bequest motives and the difficulties associated with RM securitization (Szymanoski et al., 2007; Bishop and Hui, 2008). Some authors have argued that the high cost of financing RMs, as well as the low monthly payments resulting from the conservatively low principal limit factor (hereafter PLF) set by the Department of Housing and Urban Development (hereafter HUD),[4] could be major factors adversely affecting elders' demand for RMs (Mayer and Simons, 1994; Redfoot, Scholen and Brown, 2007; Pu, Fan and Deng, 2013). Some studies show that the PLF may not be a reasonable value for an RM contract (Weinrobe, 1987). Several studies indicate that

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even though their house wealth exceeded £100,000.

[3] The most common type of reverse mortgage loan is the HECM, insured by the Federal Housing Administration. HECMs account for over 90% of all reverse mortgage loans originated in the U.S. market.

[4] The HUD website provides more information about the RM program. It can be found at: http://search.usa.gov/.

the mortgage insurance premium for HECM loans is too high, justifying a reduction in the front-end charge (Chen, Cox and Wang, 2010; Ji, 2012). In other words, these authors are suggesting that the PLF should be increased under the current insurance system.

An optimal PLF is important for the development of the RM market. Too low a PLF is unfavorable for RM borrowers, because they then need relatively high collateral to obtain the same payment form an RM. For example, a PLF less than 0.5 means that the actual house value is less than half its market value for the purpose of calculating the RM payment. This decreases elders' willingness to participate in the RM program and restricts the development of the RM market. In this paper, we use economic-based models to illustrate the calculation of an optimal PLF. Our numerical data prove that the current PLF is indeed too low.

In describing how to estimate an optimal PLF, it is necessary first to note the function of a PLF. Since RM contracts have non-recourse provisions, the collateral (the house) is the only source that lenders have for reclaiming the non-recourse debt. The lender may incur a loss from the crossover risk, defined as the risk for lender when the RM balance exceeds the value of the property at termination of the RM contract. Analysis of this risk is particularly important if elderly borrowers choose a tenure-payment plan, because in this case they receive monthly payments from the lender for as long as they live while continuing to occupy the property as their primary residence.[5]

Lenders usually use two ways to protect themselves against losses resulting from crossover risk: the PLF and mortgage insurance. From the actuarial viewpoint, the present value of all

[5] Borrowers' options can generally be classified as follows: (1) receive a lump sum cash payment up front; (2) receive predetermined monthly cash payments from an annuity as long as the house is the primary residence (tenure plan); (3) receive predetermined monthly cash payments from an annuity for a fixed period of time determined by the borrower (term plan); (4) receive mortgage proceeds, either in unscheduled payments or installments, at times and in amounts of the borrower's choosing until the line of credit is exhausted (line of credit plan); or (5) a combination of some or all of the above.

payments should equal an upper limit of principal at the origination time of the RM. This upper limit is calculated by multiplying the initial value of the house by the PLF value. This means that lenders can use the PLF to control the payments for an RM contract. According to the theory of risk management, limits on the payment advances can restrict future loan balances, thereby limiting the risk of a future loss. To mitigate the risk of the property depreciating more than is forecast, lenders usually decide the maximum initial loan balance based on a conservative PLF. Doing so makes them more confident that the property value will remain greater than the accreting loan amount for a long period of time, even if the depreciation of property is high. Therefore, one may well say that the PLF for lenders is a first buffer against loss from the crossover risk. If the conservative PLF is not sufficient to defend against this loss, lenders must resort to the second buffer, mortgage insurance. This insurance covers any shortfall in lenders' recovery from the sale of property, in theory eliminating their collateral risk.

Nowadays, PLF values in the U.S. can be obtained from a table published by HUD. Szymanoski (1994) advocated a model for determining a fair value for a PLF defined as the highest possible initial loan-to-value (LTV) ratio. The calculation of this PLF is based on the assumption that the present value of the total received insurance premium covers the current value of the insurer's expected losses. PLFs calculated using the model in Szymanoski (1994) are nearly equal to the values shown in the HUD table. Many researchers have referred to Szymanoski (1994) in discussing this issue as related to RMs (Tse, 1995; Piggott and Mitchell, 2004; Chen, Cox and Wang, 2010; Ma, Zhang and Kannan, 2011; Fan and Deng, 2013).

PLF estimates based on Szymanoski (1994) rely on a fully hedged crossover risk for the lender. In other words, the lender can shift all its possible losses resulting from crossover risk to the insurer, thus incurring no loss from the RM contract. However, in this case, a PLF calculated from the method of Szymanoski (1994) is too conservative, because it considers only the hedge for the insurer. As noted above, the PLF is also the buffer against loss from the crossover risk. We argue that lenders need not in fact fully hedge the crossover risk. Therefore, they can seek greater profits by setting a higher PLF.

A high PLF has several advantages. First, as is well known, the increased loan amounts resulting from a large PLF can earn the lender greater profit from the interest rate spread. Second, if the PLF increases, the payments that borrowers receive are also increased, thereby causing elders' participation rates in the RM program to rise. Third, some authors have shown how much benefit is obtained from portfolio diversification depends on how much the number of pooled RMs rises (Pu, Fan and Deng, 2013). Such an increase can be expected to raise the amounts the elderly borrow, because through diversification lenders can considerably attenuate their collateral and longevity risks. Thus, although lenders may increase their probability of loss on individual loans by raising the PLF, their overall loss can be effectively diversified through a larger portfolio if they make more of these loans.

The determination of an optimal PLF is important for participants in the RM market. The goal of this study was to come up with a way to determine an optimal PLF based on two general economic assumptions: one that assumes the lender breaks even and one that assumes the lender makes maximum profit. To determine the breakeven point, the optimal PLF is calculated based on the assumption that the expected income from the interest rate spread equals the expected average loss from the crossover risk. If this assumption is met, lenders get zero profit; in other words, they break even. On the other hand, if the PLF is set to less than this optimal level, lenders will likely earn a profit by providing RMs. Even if the lenders just break even, PLFs determined by this method can help the government develop the RM market, because such PLFs offer the maximum benefit to the borrower, provided there is no loss for the lender. Thus, if the lender is a government-sponsored organization, for which profit-making is not an objective, it may be more appropriate to cap the probability of a real loss rather than to focus on profit (Diventi and Herzong, 1991). The optimal PLF is better to be determined at the breakeven point.

In the scenario of the model of maximum profit, the optimal PLF is determined under the assumption that the marginal expected profit equals the marginal expected loss. From the general economic theory, one can assure that the lender is getting the maximum profit from its RMs when using this model.

When determining an optimal PLF, it is necessary to estimate the expected average loss from the crossover risk. Several studies have demonstrated that maximum loan amounts and maximum annuity payments are influenced by the following crucial factors: house price volatility, the remaining life of the elder, the loan interest rate, and the risk-free interest rate (Lee, Wang and Huang, 2012; Pu, Fan and Deng, 2013). These factors are all related to the interest risk, the collateral risk and the longevity risk.

Studies on the pricing of RM contracts or crossover risk have either used periodic life tables, thus neglecting the dynamics of mortality rates (Weinrobe, 1988; Szymanoski, 1994; Chinloy and Megbolugbe, 1994; Tse, 1995; Zhai, 2000), or they have ignored the inherent dynamics of interest rates (Chinloy and Megbolugbe, 1994; Szymanoski, 1994; Chen et al. 2010; Li et al., 2010; Pu, Fan and Deng, 2013). Only a few authors have discussed the effects of the dynamics of mortality rates and interest rates on the amount of the loan, the annuity payments, or the PLF (Diventi and Herzong, 1991; Szymanoski, 1994; Pu, Fan and Deng, 2013).

Recently, economic variables such as interest rate and house price have become more and more variable. In addition, elders are living longer than ever before because of advances in medicine. Thus the interest risk, the collateral risk and the longevity risk have been increasing more and more.

In this paper, we describe a well-designed model for calculating an appropriate PLF that incorporates simultaneously the three main risks: the interest risk, the collateral risk and the longevity risk. We assume that the interest rate follows the extended Vasicek model and that the house price follows the geometric Brownian motion process that is usually applied by the Federal Housing Administration (FHA). The longevity risk is measured using a Gompertz-type survival probability density function.

To evaluate the effects of these risk factors, and to further describe how changes in the optimal PLF respond to them, we also report sensitivity analyses. Our model provides valuable information that can help policy implementers and market participants determine an optimal PLF and manage RM risks.

This paper is organized as follows. In Section 2, we specify the valuation framework for calculating the expected loss from the crossover risk, including specifications for the interest risk, the collateral risk and the mortality risk. In this section, we also show how to determine the optimal PLF based on the traditional model of Szymanoski (1994) and the economic models, assuming a breakeven point and maximum profit. Results of a numerical example applying the model are presented in Section 3. We first describe how to calculate the optimal PLF for an uninsured RM, in which case the lender uses the PLF as the only buffer against losses; then we describe the calculation for an insured RM. Section 4 gives the results of sensitivity analyses aimed at assessing how sensitive PLFs are to changes in the parameters related to the longevity risk, the collateral risk and the interest risk. Finally, in Section 5 we offer concluding remarks.

2. Descriptions of the Models

This section includes four subsections. In Subsection 2.1, we introduce the basic framework of the current RM system. In Subsection 2.2, we present a formula for calculating the expected loss from the crossover risk, including specifications for the mortality risk, the interest rate process and the house price process. Our model for determining the PLF following from the traditional model of Szymanoski (1994) and the economic models are respectively shown in Subsections 2.3 and 2.4.

2.1 Basic RM model based on the current system

Let the elder's age when joining the RM program be t_0 years; in the current RM system, $t_0 \geq 62$.[6] Assume the elder owns a house satisfying all FHA's property standards and floor requirements. Let the house value be $h(0)$ at the time of RM origination. To participate in an RM program, the elder must pay the initial costs ($D(0)$), which include the organization cost, third-party charges and the upfront mortgage insurance premium. According to the non-recourse provision, the elder need not repay the interest and principal during the RM period; however, the elder is charged a monthly servicing fee. Assume the lender sets aside, from the borrower's principal limit, the present value of the total monthly servicing fees ($S(0)$), which is calculated from closing until the borrower would reach age 100. We denote the mortgage interest rate as r_C and the mortgage insurance premium as r_I. Letting r_{RM} be the compounding rate for an RM, we have $r_{RM} = r_I + r_C$.

The net principal limit ($NPL(0)$) is defined as:

[6] See the HUD website: http://search.usa.gov/.

$$NPL(0) = PL(0) - S(0) - D(0)\,, \tag{1}$$

where $PL(0)$ is the principal limit for the RM; $PL(0) = \min(h(0), H) \times \xi$, where ξ is the PLF value provided by the HUD table; H is the maximum HECM loan amount allowed by the HUD regulations (defined as the mortgage limit); $\min(h(0), H)$ is the maximum claim amount.

The first term on the right side of Equation (1) is the house value at the time the borrower signed up for the RM program. The last two terms are the participation costs for the borrower. We let $PL(0) = \omega h(0) \times \xi$, where $\omega = \min(1, \frac{H}{h(0)})$, an exogenous variable. This gives us:

$$NPL(0) = h(0)(\omega\xi - \eta)\,, \tag{2}$$

where η is the total cost ratio of the RM. Let η be a percentage of the initial house value. Thus, we have $\eta = \frac{S(0) + D(0)}{h(0)}$.

We denote the total amount of payment in the borrowers' account at time s as $\Theta(s)$. Thus, we have:

$$\Theta(s) = \sum_{t=0}^{s} V(t)(1 + r_{RM})^t\,, \tag{3}$$

where r_{RM} is the contract rate of the RM; and

$V(t)$ is the payment from the RM at time t.

Since RM borrowers will receive different cash payments according to which RM payment plan they choose, $V(t)$ can be specified in different ways in the application. The borrowers receive predetermined monthly cash payments from the annuity, the amount depending on the type of tenure payment they choose, until the RM is terminated; thus, $V(t) = V$ for $t \geq 0$, where V is the fixed monthly cash payment from the RM. If the plan is a term plan, the borrower receives predetermined monthly cash payments from the annuity for a fixed period of time (here t_1) determined by the borrower; thus, we have $V(t)=V$ if $0 \le t \le t_1$ and $V(t)=0$ if $t > t_1$. If the plan is a lump-sum plan, the borrower receives a lump sum of cash up front; thus, we have $V(0) = NPL(0)$ and $V(t)=0$ for $t>0$. As for the payment line of the credit plan, the borrower receives mortgage proceeds either as unscheduled payments or in installments, at times and in amounts of the borrower's choosing until the line of credit is exhausted. Thus, $V(t)$ is the actual withdrawal amount at time t, provided that the accumulated loan amount does not exceed the net principle limit (i.e., $\sum V(t) \le NPL(0)$).

When the borrower selects the type of RM payment $V(t)$, the debt amount $\Theta(s)$ at time s can be calculated. In the current RM market, the lump-sum payment plan is the most popular. However, if the borrower selects a tenure-payment plan, the lender probably incurs a larger loss than if the borrower selects a different type of payment plan. We therefore assume in our model that the elder chooses the tenure-payment plan.

The RM payments depend on the age of the borrower at the origination of the RM, the borrower's highest attainable age, the PLF, the initial house value, the total cost of joining the RM program and the RM interest rate. With the tenure-payment plan, the lender uses the net principal limit to decide the maximum payment at the time of origination of the contract. Let T^* be the assumed total number of payment years, $T^* = T - t_0$ in this case; T is the highest attainable age (100 in the current system).[7] From the actuarial viewpoint, we have:

$$NPL(0) = V\ddot{a}_{T^*, r_{RM}}, \tag{4}$$

where $\ddot{a}_{T^*, r_{RM}}$ is the present value of the annuity per monetary unit from time 0 to time T^*,

[7] This specification is shown on the HUD website: http://search.usa.gov/.

with interest rate r_{RM} and T^* being the total number of years until maturity.[8] From financial theory, one can obtain the following formula for the present value of the annuity with a constant contract rate γ and for a constant period s: $\ddot{a}_{s,\gamma} = \frac{((1+\gamma)^s - 1)}{\gamma(1+\gamma)^s}$.

2.2 The expected loss from the crossover risk

In determining an optimal PLF, it is necessary to consider the factors that influence the estimate of the loss expected from the crossover risk. Here we describe a model for accurately calculating this crossover risk, which has three main components: the interest risk, the collateral risk and the longevity risk. Our model assumes that the dynamics of the interest rate are specified by the extended Vasciek model, the behavior of the house price follows the log-normal distribution and the longevity risk for the elder is specified by the Gompertz survival probability density function. We illustrate these three assumptions in detail in the following subsections.

Assumption 1: Dynamics of the interest rate following the extended Vasicek model

We propose that the behavioral function of the interest rate and the house price are located in probability space (Ω, Λ_F, Q), where Ω is the state; the filtration Λ_F is generated by the information process F, $F = (F_t)_{0 \le t \le T}$; Q is the risk-neutral probability. We use the extended Vasicek model to describe changes in the interest rate $r(t)$. The evolution of the term structure is described by the dynamics of the short-term interest rate (Vasicek, 1977):

$$dr(t) = a(\bar{r}(t) - r(t))dt + \sigma_r dZ_r(t), \tag{5}$$

where

[8] With the current tenure plan, the RM disbursement is paid monthly. For simplicity, we specify this payment as yearly.

a is the adjusted speed of interest rate (a positive constant);

σ_r is the volatility of the short interest rate (a positive constant);

$\bar{r}(t)$ is the long-run short interest rate (a deterministic function of t); and

$Z_r(t)$ represents a standard Brownian motion of interest rate under the risk-neutral measure Q.

Based on Equation (5), one can obtain a closed-form pricing formula for a zero coupon bond with maturity date s at time t. Letting $P(t,s)$, $s > t$, be the price of this bond, we have:

$$P(t,s) = E[\exp(-\int_t^s r(u)du) \mid \Lambda_F] = A(t,s)\exp(-B(t,s)r(t)), \qquad (6)$$

where

$E[\cdot]$ is the expected operator;

$A(t,s) = \exp(\frac{(B(t,s)-s+t)(a^2\bar{r}(t)-0.5\sigma_r^2)}{a^2} - \frac{\sigma_r^2 B(t,s)^2}{4a})$; and

$B(t,s) = \frac{1-e^{-a(s-t)}}{a}$.

Assumption 2: Dynamics of the house price following a log-normal distribution

The house price process $h(t)$ specified under the physical measure is defined as:

$$\frac{dh(t)}{h(t)} = \mu(t)dt + \sigma_h dZ_h^P(t) , \qquad (7)$$

where $\mu(t)$ is the instantaneously expected house return, σ_h is the instantaneous volatility of the house return and $Z_h^P(t)$ represents the standard Brownian motion of the house return under physical measure.

To evaluate the crossover risk, we transfer the probability from the physical measure to the risk-neutral measure. Using the risk-neutral measure, the house price process $h(t)$ is defined as:

$$\frac{dh(t)}{h(t)} = (\mu(t) + \lambda(t)\sigma_h)dt + \sigma_h dZ_h(t) \ , \tag{8}$$

where $\lambda(t)$ is the risk premium and $Z_h(t)$ represents the standard Brownian motion of the house return under the risk-neutral measure Q. In this risk-neutral valuation framework, we have $\lambda(t) = -\dfrac{\mu(t) - r(t)}{\sigma_h}$ under the assumptions of no arbitrage and a complete market. Thus, in our valuation model, we have the following:

$$\frac{dh(t)}{h(t)} = r(t)dt + \sigma_h dZ_h(t) \ . \tag{9}$$

When the specification of the house price process includes the service flow rate, which acts as a dividend and is usually assumed to be a percentage of the house value in the literature (Kau, Keenan, Muller Ⅲ, and Epperson 1992, 1993; Kau, Keenan and Muller Ⅲ, 1993; Kau and Keenan, 1996; Ambrose, Buttimer and Capone, 1997; Bardhan, Karapandza and Urosevic, 2006), we have:

$$\frac{dh(t)}{h(t)} = (r(t) - \delta_h)dt + \sigma_h dZ_h(t) \ , \tag{10}$$

where δ_h is the service flow rate as a percentage of the house price. We let $Z_h(t)$ be correlated with $Z_r(t)$, denoted as $E[dZ_h(t)dZ_r(t)] = \rho dt$, where $E[\cdot]$ is the expected operator, and ρ denotes the correlation between the short-term interest rate and the house return.

Assumption 3: Specification of the mortality risk using the Gompertz survival probability

function

Because we are addressing mortality risk, we let the probability space be $(\Omega, \Lambda_{\Xi}, Q)$. We specify the death time τ as a non-negative random variable; $\tau \le t$ means that death occurred at or before time t. We let the jump process Ξ associated with τ be $\Xi_t = I_{\{\tau \le t\}}$, where I_{ω} is an indicator function: $I_{\omega} = 1$ for state ω (e.g., death) and $I_{\omega} = 0$ for other states. We let l $\Lambda_{\Xi} = (\Xi_t)_{t \ge 0}$ stand for the filtration generated by Ξ and we let $G(0,s)$ denote the probability of survival until time s, given that the homeowner is alive at time 0. We then have $G(0,s) = E[1 - \Xi_s \mid \Lambda_{\Xi}] = P(\tau > s \mid \tau > 0)$.

Up to the present, several well-known mortality functions have commonly been used to estimate mortality rates. Examples are the Weibull-type function for young children, the inverse-Weibull type for teenagers and the Gompertz type for adults (Brillinger, 1961; Lee and Carter, 1992; Carriere, 1992, 1994; Jasiulewicz, 1997). Because our discussion is focused on elders, we use the Gompertz function to measure the survival probability. We let $G(u)$ be the survival function at u years of age and $F(u)$ the death probability at age u, $t_0 \le u < T$; thus, $F(u) = 1 - G(u)$ and $f(u)$ is the probability density function for death at time t, where $f(u) = \dfrac{\partial F(u)}{\partial u}$. The Gompertz function is described as follows:

$$G(u) = \exp(\exp(-\frac{m}{\sigma_s}) - \exp(\frac{u-m}{\sigma_s})), \tag{11}$$

where m and σ_s are the location and dispersion parameters respectively. Therefore, the death probability density function can be described as:

$$f(u) = \frac{\partial(1-G(u))}{\partial u} = G(u)(\frac{1}{\sigma_s}\exp(\frac{u-m}{\sigma_s})). \tag{12}$$

Assuming the RM borrower is alive at time t, the conditional survival probability and conditional death probability density function are described respectively as:

$$G(t,s) = \frac{G(t+s)}{G(t)}; \text{ and} \tag{13}$$

$$f(t,s) = \frac{f(t+s)}{G(t)}. \tag{14}$$

Given the previous three assumptions, the crossover risk can be obtained under the condition of no arbitrage.

In the following sections, we first use the discrete-time approximation to compute the present value of the expected loss from the crossover risk (Broadie and Glasserman, 1997). Then we extend the model to the continuous-time frameworks. We give a reference filtration of the probability space (Ω, Λ_G, Q), where Λ_G is an enlarged filtration generated as $\Lambda_G = \Lambda_F \vee \Lambda_\Xi$. We propose that the valuation proceeds in this space.

The debt (accumulated outstanding balance) for the RM is $\Theta(s)$ at time $s = 0,1,\cdots,T^*$. According to Equations (3) and (4), we then have:

$$\Theta(s) = V(1+r_{RM})^s \ddot{a}_{s,r_{RM}}. \tag{15}$$

The lender receives an amount equivalent to the lesser of the debt value $\Theta(\tau)$ and the house value $h(\tau)$ when the borrower terminates the RM contract (e.g., because of death) at time τ, $\tau \in [1,\cdots,T^*]$:

$$\min(\Theta(\tau), h(\tau)) = \Theta(\tau) - \max(0, \Theta(\tau) - h(\tau)). \tag{16}$$

Because $\Theta(\tau)$ is the amount that should be repaid to the lender, as shown in Equation (16), the lender's potential loss (crossover risk) is:

$$\max(0, \Theta(\tau) - h(\tau)). \tag{17}$$

Equation (17) shows that the expected loss from the crossover risk is considered a put option, where the exercise price is the accumulated debt $\Theta(\tau)$ and the underlying asset value is the house value at time τ. This put option is influenced by three underlying factors: the random death time τ, the accumulated debt $\Theta(\tau)$ and the house price $h(\tau)$ (Szymanoski, 1994). Given deterministic death time s (i.e., $\tau = s$), the value of the put option is $B(0,\tau \mid \tau = s)$. If $\psi(0,\tau \mid \tau = s) = \dfrac{B(0,\tau \mid \tau = s)}{h(0)}$ is the expected loss per \$1 of the initial house value at time s, we have:[9]

$$\psi(0,\tau \mid \tau = s) = (\omega\xi - \eta) A(s, T^*, r_{RM})\ P(0,s) N(-d_2(0,s)) - e^{-\delta_h s} N(-d_1(0,s)), \tag{18}$$

where

$$A(s, T^*, r_{RM}) = \frac{e^{T^* r_{RM}}(e^{s r_{RM}} - 1)}{e^{T^* r_{RM}} - 1};$$

$$d_1(0,s) = \frac{-\ln(\omega\xi - \eta) - \ln(A(s, T^*, r_{RM})) - \ln(P(0,s)) - \delta_h s + \frac{1}{2} v^2(0,s)}{v(0,s)};$$

$$d_2(0,s) = d_1(0,s) - v(0,s);$$

$$v^2(0,s) = \int_0^s |\sigma_h(u) - b(0,u)|^2\ du;$$

[9] See Appendix A.

$$\sigma_h(u) = \sigma_h\sqrt{u}\text{ ; and }\ b(0,u) = \frac{\sigma_r}{a}(1-e^{-au})\ .$$

The above specification means that $\psi(0,\tau \mid \tau = s)$ is obtained under the assumption of a deterministic death time. To extend the above framework to a model with a stochastic death time, we assume the elder will live to a maximum age of 100 years. There is a probability of death at each time point before this maximum death age. Let T^* be the stochastically distributed number of years that the elder survives, starting with the age at joining the RM program and ending with the maximum age of 100 years. For each death time we can obtain an expected average loss $\psi(0,\tau \mid \tau = s)$; the set is defined as $\psi(0,\tau \mid \tau = 1)$, $\psi(0,\tau \mid \tau = 2)$, $\cdots$, to $\psi(0,\tau \mid \tau = T^*)$. If $\psi(0)$ is the expected loss given a random death time, we have:

$$\psi(0) = E[\psi(0,\tau)]\ . \tag{19}$$

We can obtain the value for $\psi(0)$ if the conditional probability density function for the death time is specified. As shown by Assumption 3, $f(0,s)$ is the conditional probability density function at the time of death. Accordingly, we have:

$$\psi(0) = \sum_{s=0}^{T^*} \psi(0,\tau \mid \tau = s) \times f(0,s)\ . \tag{20}$$

2.3 Principal limit factors calculated from the traditional model of Szymanoski (1994)

To reduce the crossover risk, the lender discounts the elder's house value by the value of the PLF in calculating the maximum loan balance at the origination time. We now illustrate the traditional model from Szymanoski (1994) that is used to calculate the prevailing PLF. Szymanoski (1994) defines the PLF as the highest initial loan-to-value ratio for which the insurance premium covers the expected loss from the future claim. Calculation of the PLF is based on the following assumption: the present value of the expected loss from the pooled RMs equals the present value of the expected mortgage premiums to be collected from the pooled RMs:

$$\sum_{s=0}^{T^*} E[L(s)(1+r(s))^{-s}] = \sum_{s=0}^{T^*} E[MIP(s)(1+r(s))^{-s}], \tag{21}$$

where $L(s)$ is the loss from the crossover risk incurred at time s and $MIP(s)$ represents the mortgage insurance premium scheduled to be collected at time s.

Because $\psi(0)$ is the expected loss per \$1 of the initial house value, we have

$$\sum_{s=0}^{T^*} E[L(s)(1+r(s))^{-s}] = h(0)\psi(0) \tag{22}$$

There are two types of MIP: (1) the upfront mortgage insurance premium ($UMIP$) at the initial time and (2) the insurance premium paid for each period until the RM contract is terminated. We then have $MIP(0) = UMIP$ and $MIP(s) = r_I\Theta(s)$ for $s = 1,\cdots,T^*$. This leads to:

$$\sum_{s=0}^{T^*} E[MIP(s)(1+r(s))^{-s}] = UMIP + r_I \sum_{s=1}^{T^*} G(0,s)\Theta(s)P(0,s) \tag{23}$$

According to Szymanoski (1994), the equilibrium PLF (denoted as ξ^*) can be calculated based on the following assumption:

$$\psi(0 \mid \xi^*) = \eta_I + r_I(\omega\xi^* - \eta)\sum_{s=0}^{T^*} G(0,s)A(s,T^*,r_{RM})P(0,s), \tag{24}$$

where $\psi(0 \mid \xi^*)$ is the value of $\psi(0)$ given ξ^*, and η_I is the upfront mortgage insurance premium per \$1 of the initial house value.

For a lender, this calculation of ξ^* is based on a fully hedged estimate of the crossover risk, because the insurer incurs all the possible losses from the crossover risk. However, as previously mentioned, both the house collateral and the mortgage insurance protect against high potential losses due to the crossover risk. If the estimate is fully hedged, ξ^* may be too conservative to encourage development of the RM market. Thus, we propose a model for calculating the lender's optimal ξ that is based on two general economic models of the lender's situation: breaking even and maximum profit.

2.4 Determination of an optimal PLF based on economic concepts

We argue that determination of an optimal PLF should be based on two economic concepts: the lender's earnings and loss. We assume the lender makes a scheduled payment at the origination time for issuing the RM and pays the capital cost at a fixed capital cost rate (r_ξ); thus, $r_\xi = r(0) + r_I$. The accumulated capital cost of the RM program up to time s is $\Omega(s)$ for $s = 1, \cdots, T^*$. This can be represented as:

$$\Omega(s) = V(1+r_\xi)^s \ddot{a}_{s,r_\xi} . \tag{25}$$

Given the above specification, if the elder passes away at time τ, the lender receives the value of the $\Theta(\tau)$ from the elder and uses it to pay the total cost of the capital ($\Omega(\tau)$) to the depositor. Therefore, the lender's earnings obtained from the interest rate spread of the RM at the time the RM contract is terminated are denoted as $\Theta(\tau) - \Omega(\tau)$. The expected present value of these earnings is:

$$E[(\Theta(\tau) - \Omega(\tau))(1 + r(\tau))^{-\tau}] = h(0)R(0 \mid \xi), \tag{26}$$

where $R(0 \mid \xi)$ represents the expected average earnings per \$1 of the initial house value given ξ. We then have:

$$R(0 \mid \xi) = (\omega\xi - \eta)\sum_{s=0}^{T^*}(A(s,T^*,r_{RM}) - B(s,T^*,r_{RM},r_\xi))\,f(0,s)P(0,s)\,, \tag{27}$$

where

$$B(s,T^*,r_{RM},r_\xi) = \frac{\ddot{a}_{s,\xi}}{\ddot{a}_{T^*,r_{RM}}}\,(1+r_\xi)^s\,.$$

In Equation (26), $R(0 \mid \xi)$ is calculated without taking account of the possible loss from the crossover risk. If the RM is not insured, the lender faces a possible loss, as the market value of the house is less than the accumulated debt. Thus, we treat this possible loss from the crossover risk as the implied cost of the RM for the lender. We need to take this cost into account in calculating the profit from the RM. If we describe the lender's expected loss as $h(0)C(0 \mid \xi)$, where $C(0 \mid \xi)$ is the expected average loss per \$1 of the initial house value, the lender's profit from the RM it issued is:

$$\pi(0 \mid \xi) = R(0 \mid \xi) - C(0 \mid \xi)\,, \tag{28}$$

where $\pi(0 \mid \xi)$ is the expected profit per \$1 of the initial house value. For an uninsured RM, it is the lender who shoulders the expected average loss. Thus, we have $C(0 \mid \xi) = \psi(0 \mid \xi)$.

If the government wants to encourage development of the RM market, it should set the PLF as high as possible. On the other hand, if the lender is a government-sponsored organization, for which profit-making is not an objective, according to economic theory the optimal PLF should be set under the assumption that the lender breaks even (i.e., zero profit), that is, the expected average earnings equal the expected loss:

$$R(0 \mid \xi) = C(0 \mid \xi)\,. \tag{29}$$

If the RM lender is not a government-sponsored organization, it can maximize its profit by determining the PLF based on the assumption that the marginal earnings equal the marginal cost (i.e., the marginal possible loss from the crossover risk):

$$MR(0 \mid \xi) = MC(0 \mid \xi)\,, \tag{30}$$

where $MR(0 \mid \xi) = \dfrac{dR(0 \mid \xi)}{d\xi}$ and $MC(0 \mid \xi) = \dfrac{dC(0 \mid \xi)}{d\xi}$ represent the lender's marginal expected revenue and marginal expected costs respectively, both based on ξ.

If the RM contract is insured, the lender's expected loss may differ from the above. We thus suggest an approach for calculating the optimal PLF of an insured RM from the lender's standpoint. In terms of this framework, if a lender wants to maximize its profits or encourage the greatest development of the RM market, it must be willing to incur some of the loss from the crossover risk. In other words, the insurer can only shoulder the loss that corresponds to the received mortgage insurance premium. The ratio of the insurer's burden on loss can be defined as the total collected insurance premium divided by the total expected loss from crossover risk.

From the insurer's standpoint, if an increase in the PLF causes a rise in the loan amount that an RM borrower receives, both the present value of the expected loss and the present value of the expected to-be-received premium increases. In general, the growth rate is larger for the former than for the latter (as shown in Figure 3). Therefore, the present value of the expected loss is larger than what the insurer must bear if an increase in the PLF, calculated based on economic concepts. For a new PLF, the difference between the expected loss and the received premium is:

$$\Psi(0) = \sum_{s=0}^{T^*} E[L(s)(1+r(s))^{-s}] - \sum_{s=0}^{T^*} E[MIP(s)(1+r(s))^{-s}]\,. \tag{31}$$

If the lender intends to raise the PLF above ξ^*, it must be prepared to bear any potential loss $\Psi(0)$ in excess of what the insurer must bear. Therefore, for an insured RM, the lender's expected loss is denoted as $\max(0, \Psi(0))$. We then have $h(0)C(0 \mid \xi) = \max(0, \Psi(0))$. According to Equations (22), (24) and (31), this gives us:

$$C(0 \mid \xi) = \max(0, \psi(0 \mid \xi) - (\eta_I + r_I(\omega\xi - \eta)\sum_{s=0}^{T^*} G(0,s)A(s,T^*,r_{RM})P(0,s))). \qquad (32)$$

When $C(0 \mid \xi)$ is determined by Equation (32), one can use Equations (29) and (30) to determine the optimal PLF taking account respectively of the maximum development of the RM market (the lender's breakeven situation) and the lender's maximum profit.

3. Numerical Analyses

Following is a numerical example to illustrate our model.[10] We let the elder's participation age t_0 = 70, the initial house value P_0 = \$100,000, $\omega = 1$, $r_C = 10\%$ and $r_I = 0.5\%$;[11] thus, the monthly interest rate for the RM (r_{RM}) $= 10\% + 0.5\%$. In addition, we assume there is no servicing fee ($S(0) = 0$), the initial MIP is \$2,000, and the origination fee is \$2,500;[12] thus, η_I and η are 0.02 and 0.045 respectively.

Jarrow, Lando and Yu (2005) emphasize that the equivalence of the physical and risk-neutral measures holds under the assumption of a well-diversified portfolio. Because the crossover risk can be well diversified by having a large number of pooled RMs, we calculate the market price of an RM under the assumption that the physical measure is equivalent to the risk-neutral measure. All the risks can be calculated from market information. We use parameters

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[10] All values are taken from figures published by the U.S. Department of Housing and Urban Development, available at: http://search.usa.gov/search?affiliate=housingandurbandevelopment&query=4235

[11] In 2015, the initial MIP was 0.5% or 2.5%, depending on the borrower's disbursements. Over the life of the loan, the annual MIP is 1.25% of the mortgage balance. Our assumptions are only for comparative purposes.

[12] The origination fee that HUD permits for HECMs ranges from \$2,500 to \$6,000.

estimated from actual data to perform the numerical analyses.

To measure the expected average loss from the crossover risk, we estimate the parameters of the interest rate and the house price processes using a 3-month U.S. Treasury bond and the U.S. Housing Price Index (HPI) respectively. The sampling frequency is monthly and the sample period is from January 1987 to September 2010, yielding 260 data points. Table 1 presents descriptive statistics (mean, standard deviation, median, and maximum and minimum values) for the short-term interest rate and the HPI.

< Insert Table 1 Here >

We use the maximum likelihood method to estimate the parameters of the short-term interest rate and house price processes based on the data.[13] The results are shown in Table 2. The estimated parameters are: $\bar{r}$ =8.5369 $\times 10^{-12}$, a =5.7297 $\times 10^{-4}$, σ_r =1.969 $\times 10^{-3}$, $\delta_h = 2.4637\times 10^{-7}$, σ_h =0.0728 and ρ =0.2105.

< Insert Table 2 Here >

The mortality risk is calculated from market information. The parameters in the conditional probability density function of the death time are calculated based on the U.S. Life-Table for 2005. According to this table, the oldest survival age for an RM borrower is 100 years; thus, the survival period (T^*), defined as from the elder's age at participation onset (70) to the maximum attainable age (100), is 30 years. The estimated parameter values for the Gompertz survival probability density function are m = 87.46 and σ_s = 5.1176.[14] We use these two parameters to estimate the conditional expected death probability for an RM borrower who is older than 70

[13] See Appendix B.
[14] The estimation method is shown in Appendix B.

years when joining the RM program.

< Table 3 Insert Here >

Based on the previous specifications, the basic parameter values are as follows: $t_0 = 70$ years, $\omega = 1$, $r_C = 10\%$, $r_I = 0.5\%$, $\eta = 0.045$, $\eta_I = 0.02$, $r(0) = 0.04$, $h(0) = \$100{,}000$, $\bar{r}$ =8.5369×10^{-12}, a =5.7297×10^{-4}, σ_r =1.969×10^{-3}, $\delta_h = 2.4637 \times 10^{-7}$, σ_h =0.0728 and ρ =0.2105, $T^* = 30$ years, m =87.46 and σ_s =5.1176.

In Figure 1, we show the relationship between the expected average loss from the crossover risk and the deterministic death age based on the above parameters. As shown in Figure 1, the expected average loss is an increasing convex function of the borrower's death age. That is, the longer the elder lives, the greater the lender's potential loss, which is attributable to the increasing crossover risk.

< Insert Figure 1 Here >

Figure 2 shows how to determine the optimal PLF for an uninsured RM contract, conditional on either the lender making maximum profit or zero profit (the breakeven point). It displays the expected average earnings obtained from the interest rate spread ($R(0 | \xi)$), the lender's expected average loss from the crossover risk ($C(0 | \xi)$) and the expected profit ($\pi(0 | \xi)$). The y-axis represents the lender's expected average earnings or expected average losses, and the x-axis represents the PLF values (ξ). The dotted line represents the expected average earnings calculated from Equation (27); the unbroken line gives the lender's expected average losses calculated from Equation (24); the dashed line represents the expected average profits, defined as the differences between the lender's expected average earnings and expected average losses. If the lender chooses to base the optimal value of the PLF on its maximum profit, Equation (30) can be used to determine this optimal value. Given the equilibrium point as the highest expected profit, we calculate the optimal PLF in our example to be 0.4451. As shown in Table 4, it follows from this value that the lender's expected average earnings, expected average loss, and expected profit are 0.2780, 0.0695, and 0.2085 respectively. On the other hand, if the lender chooses to base the optimal PLF for an uninsured RM contract on the assumption of zero profit (the breakeven situation), this optimal value is located at the breakeven point for the lender ($R(0 \mid \xi) = C(0 \mid \xi)$); in our example, this value is 0.8984. As shown in Table 4, it follows from this value that the lender's expected profit is zero, and its expected average earnings and expected average loss are both 0.5779.

< Insert Figure 2 Here >

< Insert Table 4 Here >

We now show how to calculate the optimal PLF for an insured RM. Figure 3 illustrates, for different PLF values, how to obtain the expected average loss from the crossover risk and the expected premium collected for each $1 of the initial house value. Note that the expected average loss is a convex function that is positively related to the PLF values; in other words, the expected collected premium increases as the PLF increases. These increases are caused by the increasing crossover risk, which in turn increases the expected premiums. According to the model in Szymanoski's (1994), the fair PLF is the point at which the lines for the expected average loss and the expected insurance premium cross in Figure 3; in our example, this value is 0.386. If the actual PLF is less than the fair PLF, the insurer may reap a profit. Moreover, Figure 3 tells us that the increase in the expected premium is less than the increase in the expected average loss, because the PLF value exceeds the value at the crossover point in the figure. Thus, one can conclude that the insurer may incur a loss if the actual PLF is larger than the fair PLF. Finally, Table 4 shows that when the PLF is set at 0.386 in our example, the lender's expected average earnings, expected average loss and expected profit are 0.2486, 0 and 0.2486 respectively.

< Insert Figure 3 Here >

Figure 4 shows how to determine the optimal PLF for an insured RM contract assuming maximum profit and zero profit (breakeven situation) for the lender. If the PLF is larger than 0.386 (the fair PLF), the expected average loss, calculated from Equation (32), should be greater than zero. If the lender chooses the PLF value based on maximum profit, the optimal PLF in our example is 0.4576. This value is larger than the fair PLF determined by the traditional method, meaning that the lender should lend the RM borrower more money. According to Table 4, the PLF for an insured RM (0.4576) is higher than the PLF for an uninsured RM (0.4451). This is reasonable, because the insurer has absorbed part of the loss. Table 4 shows that if the lender sets the optimal PLF at 0.4576, the expected average earnings, expected average loss and expected profit are 0.3008, 0.0429 and 0.2579 respectively. Although the expected average loss increases due to the rise in the PLF, because the interest rate spread increases the earnings more than it increases the loss, the expected profit also increases. The optimal PLF determined by our model represents a win-win situation, because the increase in the maximum loan caused by the higher PLF increases both elders' rate of participation in the RM program and the lender's profits. Finally, if the PLF is set at 0.4576 in our example, the expected average loss incurred from the crossover risk is 0.0934 and the expected premium received by the lender is only 0.0506. In other words, the insurer's burden ratio of loss is 54.18% (0.0506/0.0934). These results can help lenders and insurers set a reasonable PLF for this win-win situation.

< Insert Figure 4 Here >

Figure 4 also shows the optimal PLF for an insured RM when the lender's profit is zero. In our example, the optimal PLF is 0.9437 at this breakeven point. Even though the lender has no profit that can be used to set the PLF value, the borrower has the highest PLF and thus can get more payments from the RM program, thereby greatly encouraging the advancement of the RM market. Table 4 shows that at this breakeven point the lender's expected average earnings and expected average loss are both 0.6552. Thus, the insurer's burden ratio is 11.67% (0.0866/0.7418).

It follows from the above discussion that a PLF set at a conservatively low value not only reduces the borrower's maximal loan balance but also reduces the lender's profit. Also, our numerical results imply that raising the PLF value is a practical alternative. Addressing this question is essential to understanding how lenders are protected from these losses, and its answer provides evidence that regulators can use to conduct cost-benefit analyses and design more efficient policies.

4. Sensitivity Analyses

Because of advances in medicine, elders are living longer than ever before. In addition, economic factors such as interest rates and house prices are becoming more and more variable. How much should PLFs change in response to these other changes? In this section, we report sensitivity analyses in which we calculate the fair PLF by the traditional method and the optimal PLF based on the change in the lender's maximum profit in response to these various factors. The government can use the results of these analyses to determine the most suitable policy for developing the RM market.

In Figure 5 we show the relationships between the PLFs and the different RM contract rates.

The figure demonstrates that both the fair PLF and the optimal PLF are negatively correlated with the RM contract rate. In addition, the optimal PLF is always larger than the fair PLF. However, the values of these two PLFs are almost the same if the RM contract rate is low. A change in the RM contract rate has a larger effect on the fair PLF than on the optimal PLF.

< Insert Figure 5 Here>

Figure 6 displays the relationships between the PLFs and various parameters related to the interest rate, namely, the initial interest rate, its adjustment speed and its volatility. As shown in Figure 6a, the fair PLF and the optimal PLF are positively related with the initial interest rate. The slopes of these two curves reveal that the influence of a change in the initial interest rate is greater on the fair PLF than on the optimal PLF. Figure 6b shows that changes in the adjustment speed do not affect these two PLFs to a statistically significant degree. Figure 6c shows that the fair PLF and the optimal PLF are both negatively related with interest rate volatility. Collectively, these results imply that a change in the initial interest rate has the greater effects on the fair PLF and the optimal PLF.

< Insert Figure 6 Here>

Figure 7 gives the results of the sensitivity analyses of how various parameters related to house prices, such as the initial house price, the house service flow rate, the volatility of house return, and the correlation between the interest rates and the house returns, influence the PLFs. Because the slopes of these curves are negative in Figures 7a-7c, we infer that the fair PLF and the optimal PLF are negatively correlated with the initial house price, the house service flow rate and the volatility of house return. To the contrary, the slopes of the curves are positive in Figure 7d. Therefore, the two PLFs are positively related with the correlation between the house returns and the interest rates, although not to a statistically significant degree. The results reveal the factor that has the greatest influence on PLFs is the house service flow rate.

<Insert Figure 7 Here>

Finally, Figures 8a and 8b show that all the relationships between the two PLFs (fair and optimal) and the parameters of the survival probability density function (location and dispersion) are negative. These results tell us that in the current climate, the longer the elder is expected to live, the lower the value of the PLF should be set. This appearance is consistent with the current status of the RM market, with the longevity risk being negatively related with the maximum loan amount.

< Insert Figure 8 Here>

We argue that if the PLF increases, the payments that borrowers receive also increase, thereby causing elders' participation rate in the RM program to rise. The lender can have the benefits because of the effects of diversification resulting from an increase in the number of pooled RMs. Prior research demonstrates that the house price risk associated with mortgage portfolios can be effectively diversified across the different regions and categories (Eichholtz et al., 1995). Thus, although lenders may increase their probability of loss on individual loans by raising the PLF, their overall losses in a large portfolio can be effectively diversified if they make more of these loans. Here we illustrate a numerical example to show this argument. In our example, we use three PLFs (0.35, 0.5 and 0.65) and three values of the volatility of house return (0.5, 0.25 and 0.125) to discuss the different levels of effectiveness of the diversification of the RM portfolios. We give the following numerical example to clarify how the overall loss from an RM ($\psi(0)$) can be decreased through portfolio diversification.

< Insert Table 5 Here >

In Table 5, when $\sigma_h = 0.5$ the lender's expected losses $\psi(0)$ are 0.4353, 0.6808 and 0.9320 respectively for PLF values (ξ) 0.35, 0.5 and 0.65. That is, the lender's expected loss from individual loans is likely to increase as the PLF value increases, assuming constant volatility of house return. This implies that lenders risk increasing their losses on such loans by raising the PLF when the house price risk cannot be effectively diversified through a larger RM portfolio.

Next, we use the numerical results to illustrate the following inference: if the volatility of house return decreases, the lender's expected losses should be decreased. In view of this fact, Table 5 shows that if ξ is 0.35, the lender's expected losses $\psi(0)$ are 0.4353, 0.2150 and 0.0747 respectively when the corresponding volatilities of the house return (σ_h) are 0.5, 0.25 and 0.125. This means that lenders can reduce their expected losses if their house price risk can be decreased.

If an increase in the PLF increases elders' participation rate in the RM program, the number of pooled reverse mortgages may become sufficiently large. Thus, the volatility of the underlying property portfolio is less than that of any individual house due to the consequences of diversifying the property portfolio. Next, we clearly illustrate that having a large RM portfolio may reduce the expected loss if the house price risk can be effectively diversified. We assume that volatility of house return decreases when the number of pooled reverse mortgages becomes larger and larger because of an increase in the PLF. For example, if ξ=0.35, 0.5 and 0.65, then σ_h becomes 0.5, 0.25 and 0.125, respectively. In Table 5, $\psi(0)$=0.4353 for ξ=0.35 and σ_h = 0.5; $\psi(0)$ =0.4010 for ξ =0.5 and σ_h = 0.25; and $\psi(0)$ =0.3869 for ξ =0.65 and

$\sigma_h = 0.125$. These values reveal that even though the PLF increases, the expected loss is still reduced because of the effects of RM portfolio diversification.

Collectively, all the above numerical examples tell us that although lenders may increase their probability of loss on individual loans by raising the PLF, their overall losses can be decreased by effectively diversification with a larger portfolio if they make more of these loans.

5. Conclusions

Because the RM contract includes a non-recourse provision, lenders can incur a crossover risk if the RM balance exceeds the value of the property at termination. In the current RM system, lenders use a conservatively low PLF, determined by mortgage insurance, to protect themselves from possible losses. However, setting the PLF value so low that it adversely influences elders' participation rate to a significant degree decreases the development of the RM market, because the monthly payments the elderly borrowers receive might be too low to sufficiently improve their financial situations. In this paper, we have described a model to determine an optimal PLF.

As is well known, RM lenders usually protect themselves from losses due to the crossover risk by utilizing PLFs and mortgage insurance. Traditional researchers have always calculated the fair PLF value using the model in Szymanoski (1994). However, the PLF obtained by this method is too conservative to be optimal, because it uses only mortgage insurance to fully hedge the lender's crossover risk. We calculate the optimal PLF under two general economic assumptions: the lender breaking even and the lender obtaining maximum profit. To the best of our knowledge, this is the first paper to describe determination of the optimal PLF under these two economic circumstances.

To determine the optimal PLF, we must first reasonably model the crossover risk. We incorporate in our model the three main risks associated with RMs: the longevity risk, the collateral risk and the interest risk. Previous models have rarely incorporated these simultaneously. In our model, we use a Gompertz-type measure for the longevity risk; the interest rate and the house price are assumed to follow the extended Vasicek model and the geometric Brownian motion process, respectively.

Our numerical analyses and sensitivity analyses support the conclusion that our model has the following four useful policy implications for market participants and government policymakers. For the government, RMs can be a good way to improve the financial circumstances of an aging population and alleviate the government's financial burden. In view of this, if the government's primary policy goal is to develop the RM market, we suggest that the optimal PLF be calculated under the assumption of the lender breaking even. In our numerical example, this optimal PLF is 0.9438. Although the lender makes no profit in this case, the increase in the monthly payments that elders receive increases their desire to participate in the RM program, thereby greatly encouraging the development of the RM market. If the participation rate rises, the beneficial effects of portfolio diversification on the collateral risk and the longevity risk can considerably attenuate lenders' and insurers' potential losses from a crossover risk. Moreover, lenders also can issue RM mortgage-backed securities (e.g., HECM mortgage-backed securities, HMBSs) if the RM portfolios become large enough. These securities not only can provide capital and liquidity for lenders and insurers, but they also can increase their profits. Referring to these discussions should help governments determine optimal policies for popularizing RMs.

As for lenders, determining PLFs on the basis of a fully hedged risk, which is currently how it is done, is too conservative to increase profits in the RM market. Determining the optimal PLF under the assumption of maximum profit is better than the current method if the lender's policy is to seek maximum profit. Although the expected average loss increases due to the rise in the PLF, the interest income increases the earnings more than it increases the loss, and the expected profit also increases. For this reason, the optimal PLF determined by our model (as shown in our numerical example) leads to an increase in both the RM borrower's maximum loan balance and the lender's profit. Such an optimal PLF is a win-win for the elder and the lender.

Thirdly, according to our previous discussion, a PLF set at a conservatively low value not only reduces the lender's profit but also decreases the borrower's maximal loan balance, thereby lowering elders' desire to participate in the RM program. Our numerical results reveal that although lenders may increase their expected losses on individual loans by raising the PLF, their overall losses can be effectively diversified through larger portfolios if they make more of these loans. This implies that raising the PLF value is a practical alternative. Our results regarding the optimal PLF when lenders break even or achieve maximum profit should help policymakers set reasonable PLFs that are beneficial to lenders, RM borrowers and insurers.

Finally, the information provided by the results of our sensitivity analyses can help policymakers and market participants modulate a reasonable PLF responsive to changes in the economic situation and the relevant risks. For example, these analyses demonstrate that PLF values are negatively correlated with the volatility of the house return and the volatility of interest rate. Policymakers should increase (decrease) the PLF when the interest rate risk and the house price risk decrease (increase). Modulating PLFs by the economic situation and the relevant risks is more reasonable than the current PLF obtained from a table published by HUD and should help increase participation in the RM market.

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Appendix A:

According to Equation (17), the expected loss from the crossover risk is a put option with underlying asset $h(s)$ and strike price $\Theta(s)$. Based on the option pricing theory, the expected value of a put option, denoted as $B(0,\tau \mid \tau = s)$, can be expressed as follows:

$$\begin{aligned} B(0,\tau \mid \tau = s) &= E[\exp(-\int_0^s r(u)du)(\Theta(\tau) - h(\tau))^+ \mid \Lambda_F] \\ &= P(0,s)\Theta(s)N(-d_2(0,s)) - h(0)e^{-\delta_h s}N(-d_1(0,s)), \end{aligned} \quad \text{(A1)}$$

where

$P(0,s) = E[\exp(-\int_0^s r(u)du) \mid \Lambda_F]$, is the price of a zero coupon bond;

$$d_1(0,s) = \frac{\ln(\frac{h(0)}{\Theta(s)}) + (-\ln(P(0,s)) - \delta_h s + \frac{1}{2}v^2(0,s)}{v(0,s)}; \text{ and}$$

$d_2(0,s) = d_1(0,s) - v(0,s)$.

The debt amount can then be rewritten as follows:

$$\begin{aligned} \Theta(s) &= V(1+r_{RM})^s \ddot{a}_{s,r_{RM}} = (\omega\xi h(0) - S(0) - D(0)) \times \frac{1}{\ddot{a}_{T,r_{RM}}} (1+r_{RM})^s \ddot{a}_{s,r_{RM}} \\ &= (\omega\xi - \eta)h(0) \times \frac{\ddot{a}_{s,r_{RM}}}{\ddot{a}_{T^*,r_{RM}}} (1+r_{RM})^s. \end{aligned} \quad \text{(A2)}$$

In a continuously time framework, we have $(1+r_{RM})^s = e^{r_{RM}s}$. In addition, the present value of the annuity is $\ddot{a}_{s,r_{RM}} = \dfrac{(e^{r_{RM}\, s} - 1)}{r_{RM}\ e^{r_{RM}\, s}}$. Thus, we have

$$\Theta(s) = (\omega\xi - \eta)h(0) \times A(s, T^*, r_{RM}) . \tag{A3}$$

Therefore, we have:

$$d_1(0,s) = \frac{-\ln(\omega\xi - \eta) - \ln(A(s, T^*, r_{RM})) - \ln(P(0,s)) - \delta_h s + \frac{1}{2} v^2(0,s)}{v(0,s)} . \tag{A4}$$

Let $\psi(0, \tau \mid \tau = s) = \dfrac{B(0, \tau \mid \tau = s)}{h(0)}$, the crossover risk per \$1 of housing value. For simplicity, we let the volatility of house return be constants. We then have:

$$\psi(0, \tau \mid \tau = s) = (\omega\xi - \eta) A(s, T^*, r_{RM})\ P(0,s) N(-d_2(0,s)) - e^{-\delta_h s} N(-d_1(0,s)) .$$

This is the Equation (18) in the paper.

Appendix B

We firstly show how to estimate the parameters of the interest rate and housing processes, employing the maximum likelihood method. The jointed probability density function of two Wiener processes is expressed as follows:

$$f(x(t)\,|\,\psi) = \frac{1}{2\pi\sqrt{|V|}}\exp(-\frac{1}{2}(x(t) - x_{\mu}(t))'V^{-1}(x(t) - x_{\mu}(t))),$$

where $x(t) = [r(t) \quad r_h(t)]'$, represents a vector for the real data for the interest rate and the change rate of housing price, respectively. $x_{\mu}(t)$ and V represent their mean and variance-covariance matrix respectively. According to Equations (5) and (10), we have

$$x_{\mu}(t) = [r(t-1) + a(\bar{r} - r(t-1))dt, \quad (r(t) - \delta_h - \frac{1}{2}\sigma_h^2)dt], \text{ and } V = \begin{bmatrix} \sigma_r^2 & \rho\sigma_r\sigma_h \\ \rho\sigma_r\sigma_h & \sigma_h^2 \end{bmatrix} dt.$$

Finally, let ψ be the vector of parameters needed to be estimated; we have

$$\psi = [\bar{r} \quad a \quad \sigma_r \quad \delta_h \quad \sigma_h \quad \rho]'.$$

We then define the log-likelihood function as:

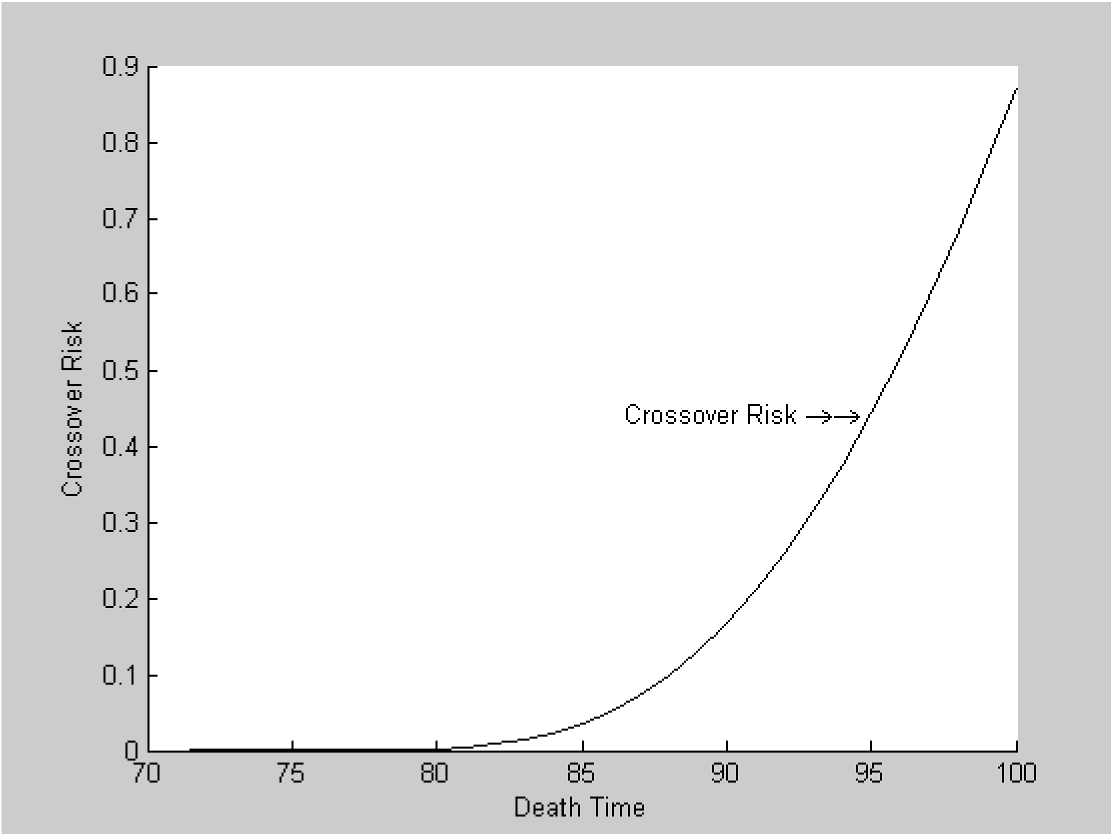
$$L(\psi) = \sum_t \log(f(x(t)\,|\,\psi)).$$

The parameter ψ is estimated if $\dfrac{\partial L(\psi)}{\partial \psi} = 0$.

In the estimation for the survival function, we minimize the mean square error between the actual data and theoretical model to obtain the location and dispersion parameter for the Gompertz survival probability. That is

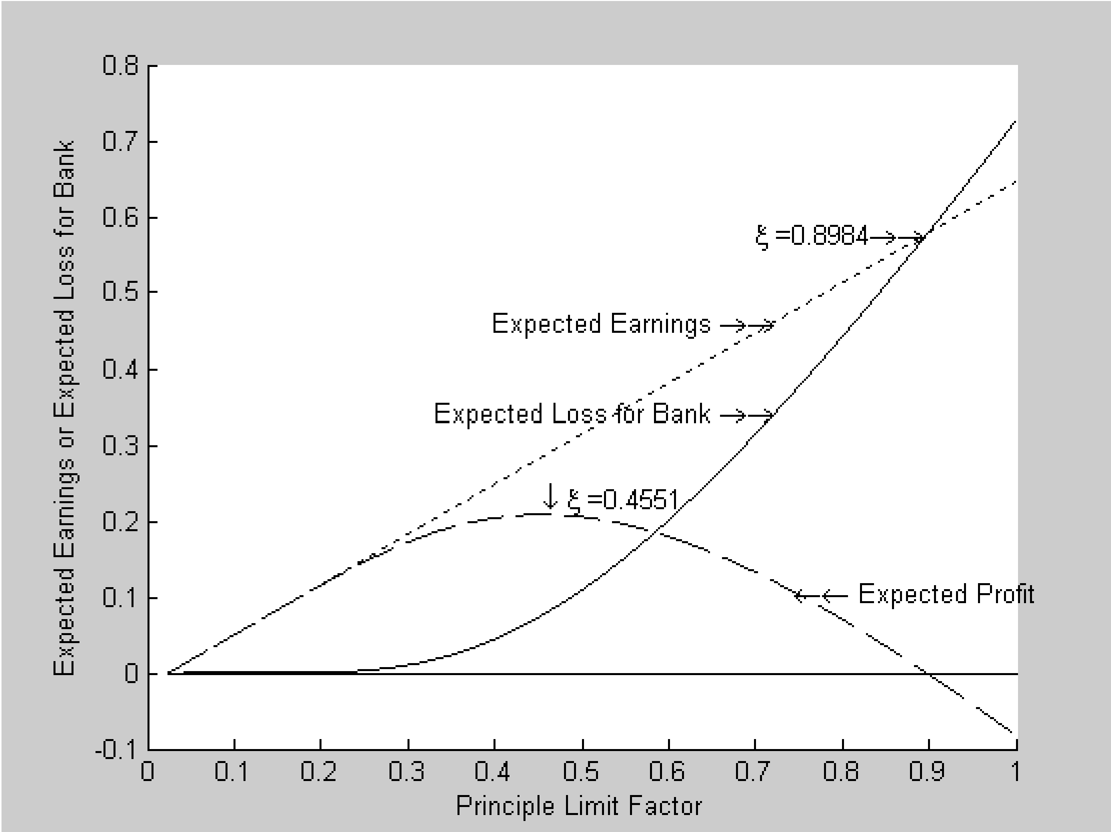
$\underset{m,\sigma_s}{Min}\ \dfrac{1}{T - t_0}\sum_{u=t_0}^{T}(\dfrac{\hat{f}(u) - f(u)}{\hat{f}(u)})^2$, where $\hat{f}(u)$ is the actual data of death probability.

Figure 1: Relationship between the average loss from the crossover risk and a deterministic age of death



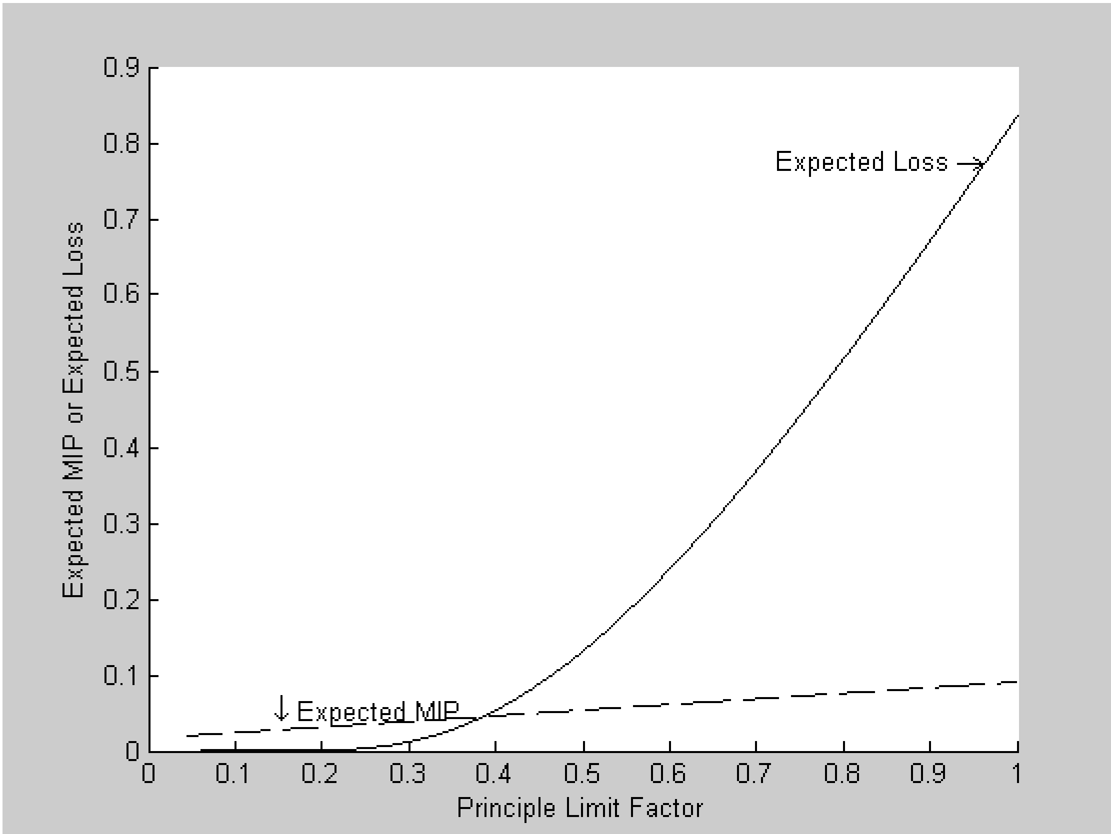
Note: The y-axis represents the expected average losses from the crossover risk, calculated from Equation (19). The x-axis represents the elders' death ages. According to our specifications, the basic parameters are: $t_0 = 70$ years, $T = 100$ years, $\omega = 1$, $r_C = 10\%$, $r_I = 0.5\%$, $\xi = 0.4551$, $\eta = 0.045$, $r(0) = 0.04$, $h(0) = \$100{,}000$, $\bar{r} = 8.5369 \times 10^{-12}$, $a = 5.7297 \times 10^{-4}$, $\sigma_r = 1.969 \times 10^{-3}$, $\delta_h = 2.4637 \times 10^{-7}$, $\sigma_h = 0.0728$ and $\rho = 0.2105$, $m = 87.46$ and $\sigma_s = 5.1176$.

Figure 2: Analysis of the optimal principal limit factors (PLFs) for an uninsured RM



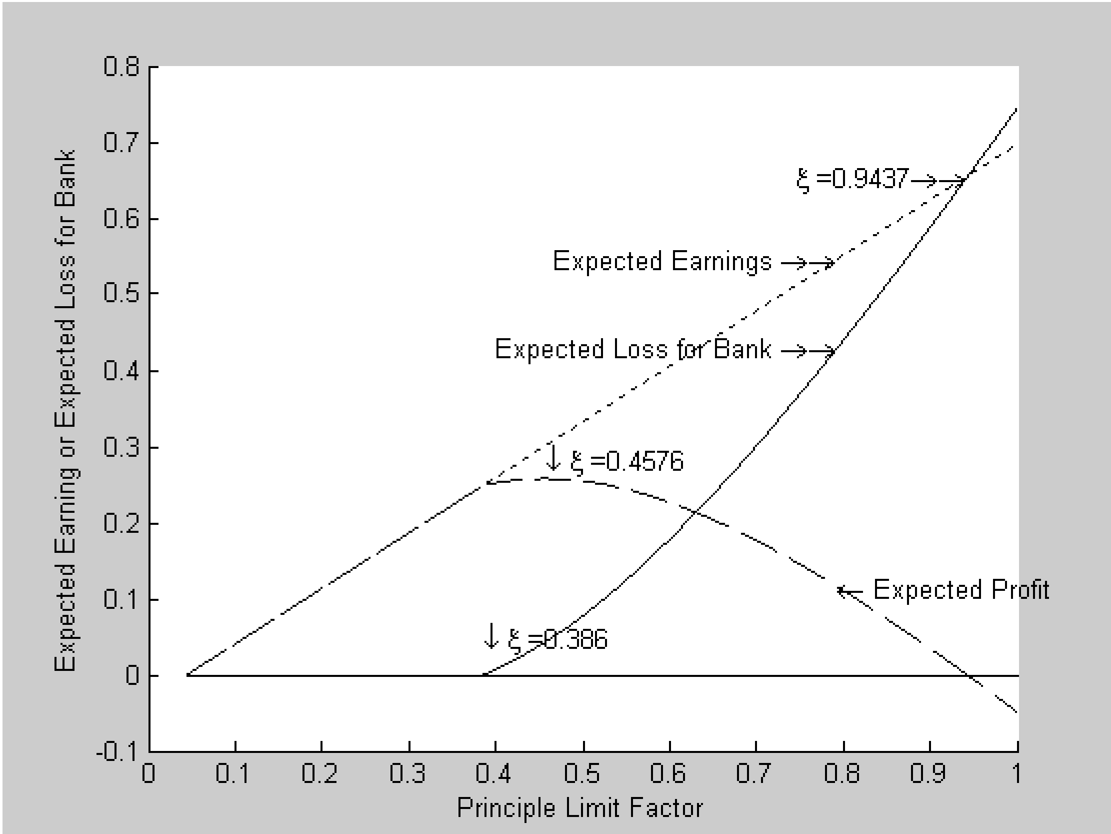
Note: The y-axis represents the expected average revenues or the expected average losses. The x-axis represents the PLF values (ξ). The dotted line represents the expected average earnings calculated from Equation (27), the solid line represents the expected average losses calculated from Equation (24), and the dashed line represents the total profits calculated as the difference between the expected average earnings and the expected average losses. The optimal PLF is 0.4451 under the assumption that the lender makes maximum profit and 0.8984 if the lender breaks even. Other definitions are presented in the note to Figure 1.

Figure 3: Analysis of the insurer's expected insurance premiums and expected average losses from crossover risk



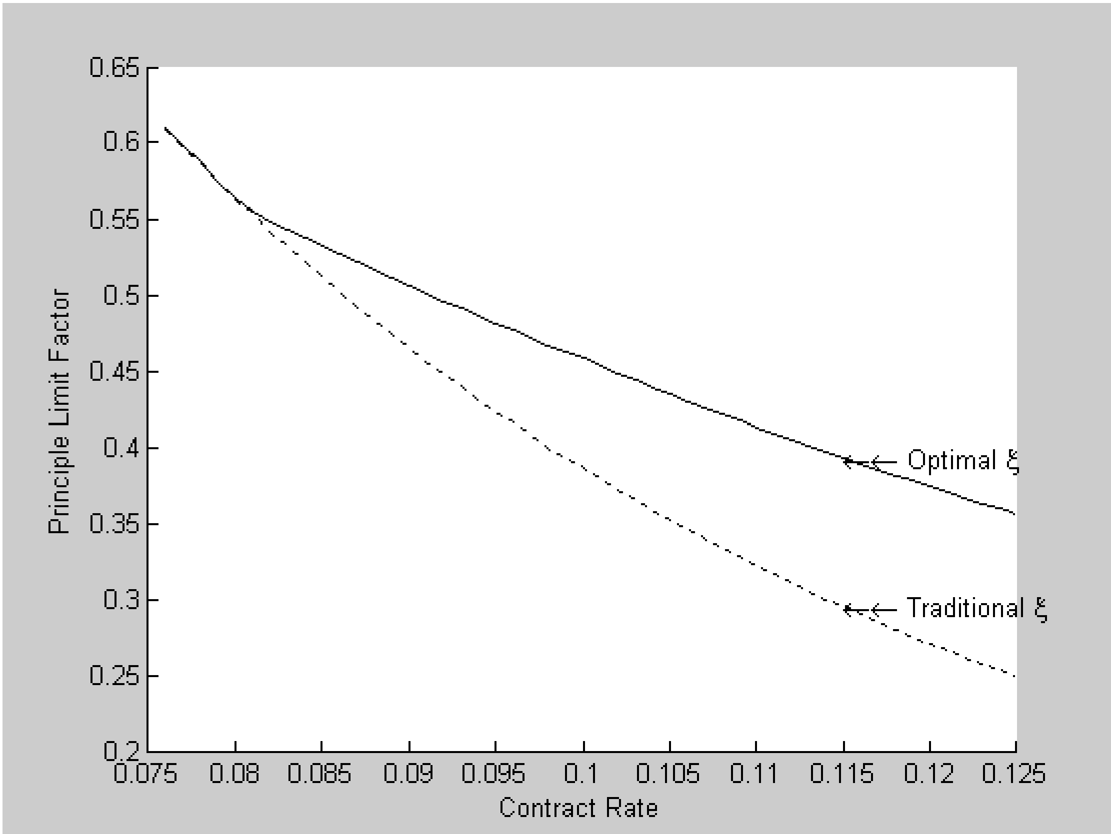
Note: The y-axis represents the lender's expected insurance premiums or expected average losses. The x-axis represents the values for the principal limit factors (PLFs), represented as ξ. The solid line represents the expected average losses calculated from Equation (22), and the dashed line represents the expected insurance premiums per monetary unit calculated from Equation (23). The crossover point of these two lines is the fair PLF, obtained by the method in Szymanoski (1994). Based on our estimated parameters for interest rates and housing prices, and the basic contract parameters in the current system, the fair PLF is calculated as 0.386. Other definitions are presented in the note to Figure 1.

Figure 4: Analysis of the optimal principal limit factors (PLFs) for an insured RM



Note: The y-axis represents the expected average earnings or the expected average losses. The x-axis represents the PLF values (ξ). The dotted line represents the expected average earnings calculated from Equation (27). The solid line represents the expected average losses calculated from Equation (32). The dashed line represents the expected profits calculated as the difference between the expected average earnings and the expected average losses. The PLF is 0.386 using the method in Szymanoski (1994). The PLF is 0.4576 if the lender makes a maximum profit and 0.9437 if the lender breaks even. Other definitions are presented in the note to Figure 1.

Figure 5: Relationship between optimal and traditional principal limiting factors (PLFs) corresponding to different RM contract rates



Note: The y-axis represents the PLF values. The x-axis represents the RM contract rates. The solid line represents the optimal PLFs assuming maximum profit for the lender. The dotted line represents the PLFs calculated by the method in Szymanoski (1994). Other definitions are presented in the note to Figure 1.

Figure 6: Sensitivity analyses for the relationship between optimal and traditional principal limit factors (PLFs) for different interest rate parameters

Figure 6a: Relationship between optimal and traditional PLFs corresponding to changes in the initial interest rate

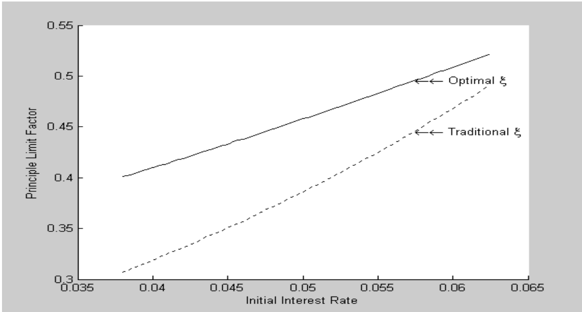


Figure 6b: Relationship between optimal and traditional PLFs corresponding to changes in the adjustment speed

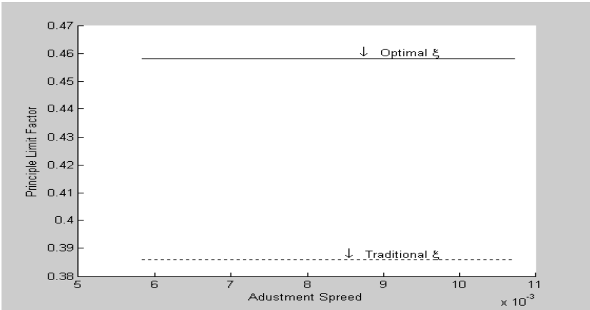
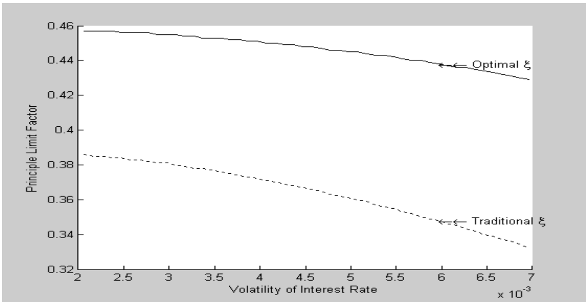


Figure 6c: Relationship between optimal and traditional PLFs corresponding to changes in interest rate volatility



Note: In Figures 6a-c, the y-axes represent the PLF values. The x-axes represent respectively initial interest rates, adjustment speeds, and interest rate volatilities. The solid line represents the optimal PLFs assuming the lender makes maximum profit. The dotted line represents the PLFs calculated by the method in Szymanoski (1994). For definitions of the other basic parameters, see Figure 1.

Figure 7: Sensitivity analyses for the relationships between the optimal traditional principle limit factors (PLFs) for the different housing price parameters

Figure 7a: Relationship between optimal and traditional PLFs corresponding to changes in the initial housing price

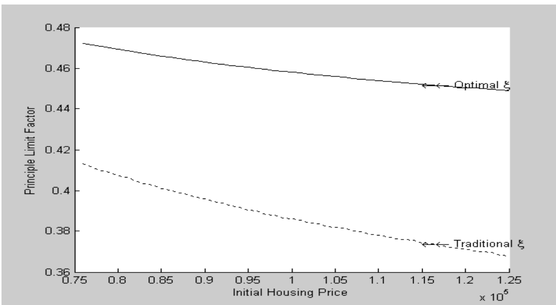


Figure 7c: Relationship between optimal and traditional PLFs corresponding to changes in the volatility of housing return

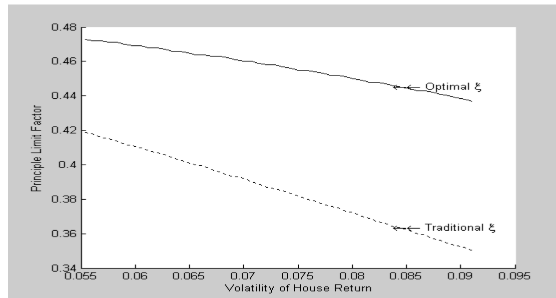


Figure 7b: Relationship between optimal and traditional PLFs corresponding to changes in the service flow rate

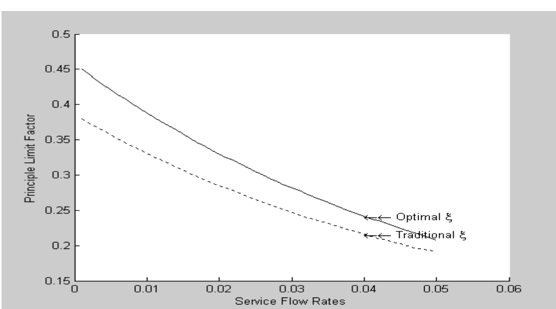
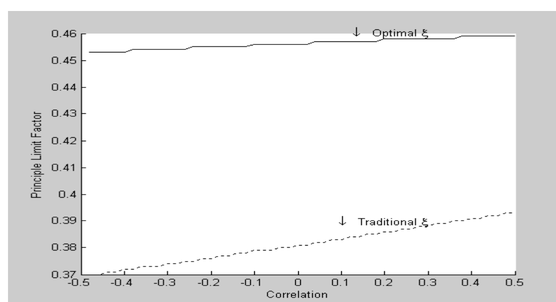


Figure 7d: Relationship between optimal and traditional PLFs corresponding to the correlation between the interest rates and the housing return



Note: In Figures 7a-d, the y-axes represent the PLF values. The x-axes represent respectively initial housing prices, the house service flow rates, the volatility of house return, and the correlation between interest rates and house return. The solid lines represent the optimal PLFs assuming lender's maximum profit. The dotted lines represent the PLFs calculated by the method in Szymanoski (1994). For definitions of the other basic parameters, see Figure 1.

Figure 8: Sensitivity analyses for the relationship between the optimal and traditional principal limit factors (PLFs) for different survival probability parameters

Figure 8a: Relationship between optimal and traditional PLFs corresponding to changes in the location parameter

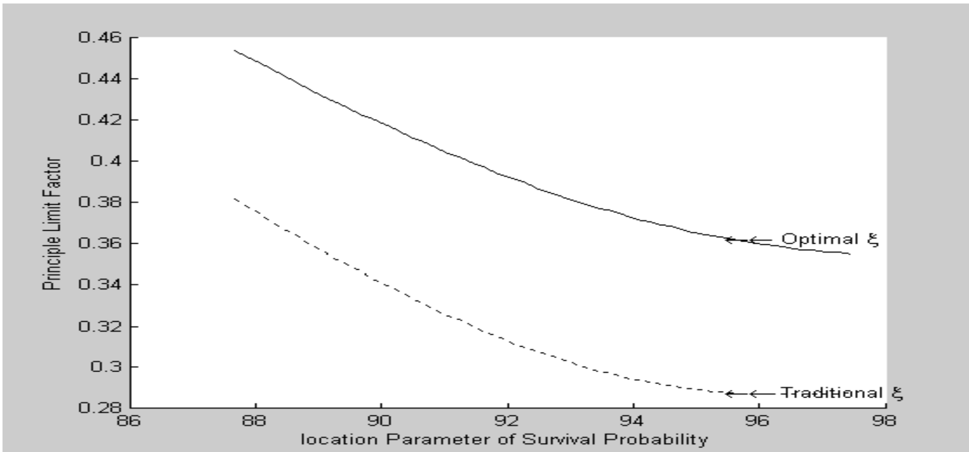
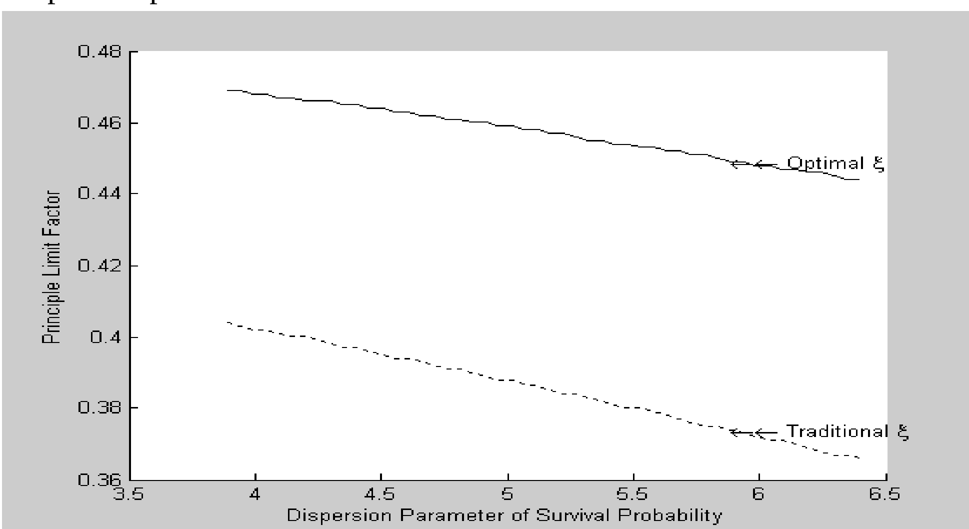


Figure 8b: Relationship between optimal and traditional PLFs corresponding to changes in the dispersion parameter



Note: In Figures 8a and 8b, the y-axes represent the PLF values. The x-axes represent respectively the location and dispersion parameters for the survival probability function. The solid lines represent the optimal PLFs assuming the lender makes maximum profit. The dotted lines represent the PLFs calculated by the method in Szymanoski (1994). For definitions of the other basic parameters, see Figure 1.

Table 1: Statistical summary of interest rates and the housing index

	Interest rate	**HPI**
Mean	0.0317	157.9800
Standard deviation	0.0201	42.2510
Maximum	0.0630	228.8800
Median	0.0353	155.1400
Minimum	0.0001	100.0000
Sample number	260	260

Note: The interest rates are for U.S. 3-month Treasury bonds and the housing index is the U.S. housing price index (HPI). The data sampling frequency is monthly and the sample period is from January 1987 to September 2010, yielding 260 total values.

Table 2: Estimated parameter values for the dynamic behavior processes of the interest rate and the housing price

	$\bar{r}$	a	σ_r	δ_h	σ_h	ρ
Estimated parameter value	8.5369×10^{-12}	0.0057297	0.01969***	2.4637×10^{-7}	0.07286***	0.2105 ***
P-value	0.5	0.2473	0	0.49998	0	4.9974×10^{-5}

Note: $\bar{r}$ is the long-run interest rate, a is the adjusted speed; σ_r is the volatility of the change of the interest rate; σ_h is the volatility of the housing return; δ_h is the house service flow rate, and ρ is the correlation between the short-term interest rate and the housing return. *** denotes significance at the 1% level.

Table 3: Estimated parameter values for the survival function

	m	σ_s
Estimated parameter value	87.46***	5.1176***
P-value	0	0

Note: m and σ_s are the location and dispersion parameter respectively. The definitions of each parameter are shown in Equation (11). *** denotes significance at the 1% level.

Table 4: The optimal principle limit factors and profits for uninsured and insured RM

		Traditional Method	**Maximum Profit**	**Breakeven (Zero Profit)**
Uninsured RM	Optimal PLF		0.4451	0.8984
	Expected Earnings		0.2780	0.5779
	Expected Loss		0.0695	0.5779
	Expected Profit		0.2085	0.0000
Insured RM	Optimal PLF	0.3860	0.4576	0.9437
	Expected Earnings	0.2486	0.3008	0.6552
	Expected Loss	0.0000	0.0429	0.6552
	Expected Loss	0.0451	0.0934	0.7418
	Expected MIP	0.0453	0.0506	0.0866
	Expected Profit	0.2486	0.2579	0.0000

Note: “Traditional Method”, “Maximum Profit”, and “Breakeven” represent that the calculations are used by the method in Szymanoski (1994), the method under the assumption that the lender achieves maximum profit and the method under the assumption that the lender breaks even, respectively. The expected average profit is calculated from the difference between the expected average earnings and the expected average loss. The expected loss is the expected average loss minus the total expected mortgage insurance premium (MIP). All results are calculated based on estimated parameters for the interest rate and the housing price and the given basic contract parameters in the current system.

Table 5: Expected losses from RMs as a function of different PLFs and different house return volatilities

PLF Value	Volatility of Housing Return		
	$\sigma_h = 0.5$	$\sigma_h = 0.25$	$\sigma_h = 0.125$
$\xi = 0.35$	0.4353	0.2150	0.0747
$\xi = 0.50$	0.6808	0.4010	0.2076
$\xi = 0.65$	0.9320	0.6090	0.3869

Note: This table gives values of $\psi(0)$, representing the lender's expected losses from RMs as a function of different PLFs (ξ) and different house return volatilities (σ_h). $\psi(0)$ is obtained from Equation (20).